Additive Decomposition of Shear Strength in Cohesive Granular Media from Grain-Scale Interactions

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We study cemented granular media by introducing cohesive bonding (sliding or rolling friction and tensile strength) between grains in the framework of the contact dynamics method. We find that, for a wide range of bond parameters, the macroscopic angle of friction at the peak state can be split into three distinct terms of collisional, frictional and dilatational origins. Remarkably, the macroscopic tensile strength depends only on the bond tensile strength, and the friction angle at the peak state is proportional to the dilatancy angle which varies linearly with sliding friction.

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Cohesive granular materials are of particular interest to various fields of science and engineering such as soil mechanics, geology, and processing of fine powders and grains. Although cohesive interactions can have very different physicochemical origins, their common denominator is to freeze, up to a threshold in force or torque, the relative degrees of freedom between grains. Besides the Coulomb friction, which is also present in cohesionless materials as the main source of shear strength, well-known examples of cohesive bonding are mineral cementation, capillarity, and van der Waals forces in dense or fine-grained materials [1,2].

From a grain-scale viewpoint, an interesting and largely open issue is how local force thresholds scale up to macroscopic strength properties (friction angle and cohesion). However, the influence of local thresholds can hardly be investigated by direct experiments. The thresholds are material dependent and therefore cannot be isolated and varied methodically. A unique opportunity is offered by capillary cohesion which can be controlled by liquid content [3–5]. For example, several studies have focused on the effect of the adhesion force on the maximum angle of stability [6].

On the other hand, discrete element simulations provide a suitable tool to investigate large scale properties from local parameters. Several numerical studies have been reported focusing mostly on the compaction and flow properties of cohesive granular materials [7–11]. We are aware of few numerical studies dealing in a systematic manner with the macroscopic strength properties [12,13].

In this Letter, we present a detailed analysis of the shear strength in cohesive packs simulated by the contact dynamics method. Parametric study leads to an appealing picture depicting the precise role of each bond parameter in the cohesive behavior. The model of cohesive bonding in the simulations was designed to mimic cohesion by cementation as observed in rocks and soils. It incorporates the following features: (1) cohesive interactions are governed by shear force and torque thresholds that are proportional to the normal force, as well as a tensile force threshold [Fig. 1(a)]; (2) cohesive bonding occurs inside a small zone between grains, representing in a way the cementing material [Fig. 1(b)]; (3) contact rupture leads to an irreversible loss of tensile strength (debonding) and the contact turns to purely frictional behavior.

In the following, we attribute positive values to compressive forces. By definition, the tensile force threshold $\tau_\alpha$ is the largest tensile force that can be supported by a pair of grains along the contact normal. It is also useful to introduce a tensile stress $\sigma_\alpha \equiv \tau_\alpha / \ell$ where $\ell$ is the mean diameter of the grains in contact. For sliding friction, we use Coulomb’s law in which the shear force threshold $T_{\max}$ is given by

$$T_{\max} = \mu_s (N + F_a),$$

where $\mu_s$ is the coefficient of sliding friction and $N$ is the normal force. The offset $\mu_s F_a$ can be interpreted as the effect of adhesion forces increasing the contact area and thus the resistance to sliding.

Full cementation requires also torque transmission at the contact points. This is equivalent to a rolling resistance with possibly a torque threshold $\Gamma_{\max}$. We assume that $\Gamma_{\max}$ obeys a Coulomb-like law [14]

$$\Gamma_{\max} = \mu_r (N + F_a),$$

where $\mu_r$ is the coefficient of rolling friction. Obviously, the length scales involved in the description of rolling resistance are those of the cohesive bond. In grain-scale numerical simulations, however, it is not desirable to in-
troduce small length scales compared to grain size. In Eq. (2), the presence of \( \ell \) is simply meant to make \( \mu_r \) dimensionless. At the particle scale, \( \mu_r, \sigma_r, \) and \( \sigma_a \) represent the microscopic parameters of the model that express at this scale the underlying physicochemical properties of the contact interactions.

The kinetics of contact loss and gain is governed by a creation length \( \delta \) and an elliptic zone of weak extension located between the grains; see Fig. 1(b). The cohesive bonds are created initially between all grains such that the ratio of strains plots at different levels of confinement. The stress constructed in the stress space (see inset to Fig. 5). For each combination of bond parameters, we determined the angle of friction \( \phi_p \) (the slope of the failure line) and the cohesion \( C_p \) (intersection with the \( \tau \) axis) at the peak state. We also determined the angle of friction \( \phi_R \) for the residual state (the cohesion \( C_R \) being zero in this state), as well as the dilatancy angle \( \psi \equiv \sin^{-1}(\delta e_\rho/\delta e_\eta) \) at the peak state.

Figure 3 shows \( \phi_p \) and \( \phi_R \) as a function of the sliding friction angle \( \phi_s \equiv \tan^{-1}(\mu_r) \) for three values of the coefficient of rolling friction \( \mu_r \). Each symbol and the corresponding error bar represent the mean and extreme values of \( \phi_p \) and \( \phi_R \) for six different values of the tensile strength \( \sigma_a \). A remarkable feature of these plots is that the macroscopic friction angles \( \phi_p \) and \( \phi_R \) are independent of the tensile strength as the variability is small around the mean values. Both \( \phi_p \) and \( \phi_R \) increase with \( \mu_r \).

As \( \phi_s \) tends to zero, both angles of friction tend to a finite value \( \phi_0 \) lying between 4° and 7°. This shows that contact friction is not the only source of frictional behavior at the macroscopic scale. Energy dissipation in this “frictionless” limit can take place only as a result of inelastic collisions through unstable particle rearrangements. Dissipation by inelastic collisions being proportional to
the mean stress, its effect appears as a Coulomb-like friction law with an apparent angle of friction \( \phi_0 \).

The residual angle of friction \( \phi_R \) increases with \( \phi_s \) from \( \phi_0 \) and saturates to a value \( \phi_\infty \) that depends only on the rolling friction \( \mu_r \). The data are well fit to an exponential function:

\[
\phi_R = \phi_0 + (\phi_\infty - \phi_0)(1 - e^{-\alpha \phi_s}),
\]

with \( \alpha \approx 0.1 \). Remark that, within statistical precision, \( \alpha \) is independent of \( \mu_r \). The nonlinear dependence of \( \phi_R \) on \( \phi_s \) is consistent with other simulations reported in the past for \( \mu_r = 0 \) [19,20]. The saturation of \( \phi_R \) suggests that, as \( \phi_s \) increases, a transition occurs in the grain-scale phenomena underlying macroscopic friction. In particular, sliding and rolling appear to be the dominant deformation modes at low and high sliding frictions, respectively. It is a basic observation that the stress-strain behavior is strongly dependent on the initial value of the solid fraction. Only the so-called “critical state,” reached at large shear strains, is independent of the initial state [18]. This “critical state” is such that the stress ratio \( \sigma_f / \sigma_t \) is constant and the dilatancy angle is zero on average. Thus, the strength parameter \( \phi_R \) in the critical state should be considered as a material property.

On the other hand, the peak strength \( \phi_p \) reflects the initial compactness of the material: higher dilatancy and peak friction angles are expected for higher initial compactness of the material. To check this correlation directly from numerical data, we plot in Fig. 4(a) the dilatancy angle \( \psi \) at the peak state as a function of the difference between peak and residual friction angles \( \Delta \phi = \phi_p - \phi_R \). We observe a nice linear dependence with a slope close to 1. This indicates that the difference between peak and residual friction angles is fully explained by dilatancy. The dilatancy angle \( \psi \) for \( \Delta \phi = 0 \) corresponds to the macroscopic friction angle \( \phi_0 \) for frictionless particles (see Fig. 3). Both, \( \psi \) and \( \Delta \phi \), are nearly linear functions of \( \phi_s \) and are barely influenced by \( \mu_r \) [Fig. 4(b)]. Hence, with a good approximation, the dilatancy angle can be split into two parts:

\[
\psi \equiv \phi_0 + \Delta \phi \equiv \phi_0 + \phi_\infty - \phi_0 + k \phi_s,
\]

with \( k = 0.5 \). This remarkable correlation between friction and dilatancy angles, here verified for a cohesive granular material, is in agreement with Taylor’s model proposed for cohesionless sand and known as stress-dilatancy relation [18].

We now consider the Coulomb cohesion \( C_p \) with respect to the influence of bond parameters. For reasons that will become clear below, we focus on the “theoretical” tensile strength \( A^* = C_p / \mu_p \) obtained by extrapolating the Mohr-Coulomb envelope from the compressional regime to the tensile regime (see inset in Fig. 5) [21].

Figure 5 shows \( A^* \) as a function of the local tensile strength \( \sigma_a \). Each symbol and the corresponding error bar represent the mean and extreme values of \( A^* \) for the whole set of sliding and rolling frictions. We observe a linear dependence of the macroscopic tensile strength \( A^* \) on \( \sigma_a \):

\[
A^* = \beta \sigma_a,
\]

with \( \beta = 1.4 \). Interestingly, \( A^* \) is independent of both sliding and rolling frictions. This feature may be explained by considering the stress on a plane perpendicular to the direction of extension and remarking that the friction forces and torques from contacts oriented along different directions cancel out along this direction [5,21]. This shows that the macroscopic strength parameters have different local origins. Local friction coefficients and inelastic collisions are responsible for the macroscopic friction, whereas the tensile strength at the contact scale seems to be the unique origin of macroscopic tensile strength.

The picture arising from this parametric study is both simple and rich. Equations (3) and (4) suggest a simple and rich. Equations (3) and (4) suggest an additive partition of the friction angles as illustrated in Fig. 6. In particular, the peak friction angle \( \phi_p \) is expressed as a sum of three terms of different origins:

\[
\phi_p = \phi_0 + \phi_\mu + \Delta \phi \equiv \phi_\mu + \psi.
\]

The offset \( \phi_0 \) is a purely collisional contribution independent of rolling friction and tensile strength. The term \( \phi_\mu = \phi_R - \phi_0 \) represents a purely frictional contribution. It is an exponential function of \( \phi_s \) and saturates to \( \phi_\infty - \phi_0 \)}
which increases with rolling friction $\mu_r$. The term $\Delta \phi \equiv \psi - \phi_0$ represents the contribution of dilatancy to the shear strength and, according to Eq. (4), it is an approximately linear function of $\phi_s$.

In the same way, from Eqs. (5) and (6), the Coulomb cohesion can be expressed by a simple form:

$$C_p \equiv \beta \sigma_a \mu_p \equiv \beta \sigma_a \tan(\phi_0 + \phi_\mu + k k_s).$$

This writing shows that the Coulomb cohesion is an increasing function of all bond parameters, the basic contribution coming from the tensile strength as a multiplicative factor. It is noteworthy that the cohesion in the limit $\phi_s = \phi_0$, i.e., $C_p = \beta \sigma_a \tan(\psi)$, in the absence of this term, the packing would behave as a cohesionless material independently of the local tensile strength $\sigma_a$. The constant $\beta$ is likely to depend strongly on the distribution of cohesive bonds in the initial state. In particular, the fraction of cohesive bonds decays in inverse proportion to the solid fraction, suggesting that $\beta$ will decrease dramatically in loose packings. The same is true for the parameter $k$ relating $\psi$ to $\phi_s$. The dilatancy varies with the initial value of the solid fraction so that we expect lower values of $k$ down to zero when the solid fraction is decreased to the critical state solid fraction.

The well-defined partition of the macroscopic angles of friction for a wide range of sliding and rolling friction coefficients at the contact scale in cohesive granular media is a key finding of the present investigation. In view of the large number of independent numerical tests performed, this result provides a solid basis for further insight into the behavior of cohesive granular media. Along these lines, a more detailed analysis of the respective roles of the sliding and rolling friction with respect to the frictional term $\phi_\mu$ merits further investigation. It is also important to consider less compact configurations than those studied in this work. Finally, the evolution of granular microstructure up to and beyond the peak state is a crucial issue for understanding the scale up of bond interactions. On more general grounds, some quantitative correlations discussed in this Letter might turn out to be closely related to the underlying model of cohesive bonding. But, we believe that the partition of the Mohr-Coulomb strength parameters $\phi_p$, $\phi_R$, and $C_p$ into collisional, frictional, and dilatational parts, and the role of the local tensile strength as the unique source of the macroscopic tensile strength, are robust features of cohesive granular materials with respect to the model of cohesive bonding.

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